# QRC-ESPRIT Method for Wideband Signals

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*Abstract* – **In this paper, a new algorithm for a high resolution Direction Of Arrival (DOA) estimation method for multiple wideband signals is proposed. The proposed method proceeds in two steps. In the first step, the received signals data is decomposed in a Toeplitz form using the first-order statistics. In the second step, The QR decomposition is applied on the constructed Toeplitz matrix. Compared with existing schemes, the proposed scheme provides several advantages. First, it requires computing the triangular matrix R or the orthogonal matrix Q to find the DOA; these matrices can be computed with O($n^2$) operation. However, most of the existing schemes required eignvalue decomposition (EVD) for the covariance matrix or singular value decomposition (SVD) for the data matrix; using EVD or SVD requires much more complex computational O($n^3$) operation. Second, the proposed scheme is more suitable for high-speed communication since it requires first-order statistics and a single snapshot. Third, the proposed scheme can estimate the correlated wideband signals without using spatial smoothing techniques; whereas, already-existing schemes do not. Accuracy of the proposed wideband DOA estimation method is evaluated through computer simulation in comparison with a conventional method.**

*Index terms* – **array signal processing, uniform linear array, Wideband signal, Toeplitz matrix.**

## I. INTRODUCTION

Source localization using a sensor array is one of the key techniques in mobile communication. It plays an important role in many application fields such as radar, sonar, mobile communication, Multiple-Input Multiple-Output (MIMO) system, biomedical, and wireless sensor networks [1], [2]. Numerous classical subspace methods e.g., Multiple Signal Classification (MUSIC) [3], Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [4], and Maximum Likelihoods method (ML) [5] have been developed to estimate the DOA of the narrowband signals. Many of these narrowband methods are not applicable to wideband signal directly. The reason is that the energies of narrowband signals are concentrated in the frequency band that is relatively small compared with center frequency. The wideband signals encountered in many applications such as high-speed data communication, passive sonars, and speech signal processing . A wide range of techniques have been proposed to deal with the wideband source localization problem. The maximum likelihood extended for the wideband signal scenario in [6] and [7] provides optimal statistic properties, but it requires a multi-dimensional search and is highly non-linear. The one dimensional search method in [8] proposed for non-coherent signals requires less computational load. A class of representative high-resolution DOA estimation approaches for wideband sources when the signals are highly correlated due to multipath or reflection is the coherent signal subspace method (CSSM) in [9] and [10]. The CSSM converts the wideband signal subspaces into a predefined narrowband subspace by focusing matrices; subsequently, the narrowband subspace-based DOA estimation methods, e.g. MUSIC, can be applied. However, the CSSM method requires initial DOAs, and its performance is sensitive to these initial values. Two other classes of wideband source localization approaches are based on orthogonality of subspace. In one class the test of orthogonality of projected subspace (TOPS) [11] is referred to as a non-coherent method. The method requires singular value decomposition (SVD) to get the signal subspace of each frequency point. However, using the SVD required the computational cost of O($n^3$) operations. In the other class, the test of orthogonality of frequency subspace (TOFS) [12] has been proposed for non-coherent wideband signals . TOFS constructs the searching steering vector of every possible DOA and frequency. However, TOFS cannot resolve the desired DOAs when the signal to noise ratio (SNR) is low.

The objective of this paper is to provide an efficient algorithm to estimate DOAs for multiple wideband sources and to reduce the computational complexity when compared with the existing schemes. To achieve these objectives, the proposed method proceeds in two steps. In the first step, the received signals data is decomposed in a Toeplitz form using the first-order statistics. In the second step, the QR (no previous reference for an acronym) decomposition is applied on the constructed Toeplitz matrix. Compared with existing schemes, the proposed scheme provides several advantages. First, it requires computing the triangular matrix R or the orthogonal matrix Q to find the DOA. The computation of QR for $n \times n$ with the Toeplitz matrix [13-14] requires $O(n^2)$; whereas, the existing schemes require eigenvalues decomposition (EVD) or singular value decomposition (SVD) for an $n \times n$ cross-spectral matrix. The major drawback of SVD or EVD is that they require much more computational complexity $O(n^3)$. In addition, the proposed method is more suitable for high-speed communication first order statistics, a single snapshot, and requires computing the triangular matrix R or Q only, which further reduces the computational complexity. However, the existing method involves multi-dimensional research or high order statistics which increases the complexity and computational load even more. In addition, the proposed method is able to resolve the DOAs of highly


DOI: 01.IJCOM.02.03.32

correlated sources. This paper is organized as follows: in section II, the proposed method is introduced based on first and second order statistics. In section III, we present the simulation results. A conclusion is given in Section IV

## II. SYSTEM MODEL AND PROPOSED METHOD

Consider a symmetric uniform linear array (ULA) consisting of *M=2N+1* identical omni-directional antennas, where the distance between adjacent antennas is *d*. Consider *L* wideband signal sources from the directions $\theta_1, \theta_2, \cdots, \theta_L$. Then the received signal by the *m*-th sensor can be expressed as

$$x_m(t) = \sum_{l=1}^{L} s_l(t - \tau_m(\theta_l)) + n_m(t) \quad (1),$$

where $s_l(t)$ is the *l*-th source signal; $n_m(t)$ is the additive white Gaussian Noise (AWGN) at the *m*-th sensor; and $\tau_m(\theta_l) = (m)d\cos\theta_l / v$ is the delay, where *v* and *d* denote the propagation speed of source signals and the distance between sensors, respectively. A narrowband DOA estimation and source localization problem requires that the inter-sensor spacing should satisfy the so-called half-wavelength constraint to avoid the phase ambiguity caused by the multi-valued property of the cosine function. However, half-wavelength constraint is not valid to a multiple wideband source data model. Applying the Discrete Fourier Transform (DFT) to (1), we obtain

$$x_m(\omega_k) = \sum_{l=1}^{L} s_n(\omega_k) e^{-j\omega_k \tau_m(\theta_l)} + n_m(\omega_k) \qquad m = -N, -N+1, \cdots, 0, 1, \cdots, N \quad (2).$$

If we use the element at the center of the array as a reference point, then the $(2N+1)\times 1$ output vector can be written as

$$X(\omega_k) = [x_{-N}(\omega_k), \cdots, \quad x_0(\omega_k), \cdots, \quad x_N(\omega_k)]^T \quad (3),$$

where the superscript $(.)^T$ represents transpose. We can express $X(\omega_k)$ as

$$X(\omega_k) = \sum_{l=1}^{L} s_l(\omega_k) a(\omega_k, \theta_l) + n(\omega_k) \quad (4),$$

where $a(\omega, \theta_l)$ is the array response vector in spatial-frequency domain and is defined as

$$a(\omega_k, \theta) = \left[e^{j\omega_k \tau_{-N}(\theta)} \quad \cdots \quad 1 \quad \cdots \quad e^{j\omega_k \tau_N(\theta)}\right] \quad (5).$$

The matrix formulation of (4) can be written as

$$X(\omega_k) = A(\omega_k) s(\omega_k) + n(\omega_k) \quad (6),$$

where $s(\omega_k) = [s_1(\omega_k), \cdots, s_L(\omega_k)]^T$ and $n(\omega_k) = [n_{-N}(\omega_k), \cdots, n_N(\omega_k)]^T$ are the DTFT data vectors of the source and noise, respectively. The $(2N+1)\times L$ matrix

$$A(\omega_k) = [a(\omega_k, \theta_1), \cdots, a(\omega_k, \theta_L)] \quad (7)$$

is the array manifold matrix in the spatial-frequency domain. Equation (6) means that in the time-frequency domain, the model of wideband and nonstationary source localization is similar to the one of narrowband and stationary case at each time and frequency bin. For the first step in the proposed method, we convert the output data vector $X(\omega_k)$ into a Toeplitz Hermitian data matrix with dimensions $(N\times 1)\times(N\times 1)$. The advantage of introducing the Toeplitz Hermitian data matrix is that it has a rank that is related to the DOA of the sources irrespective of whether the sources are coherent or not. This matrix, denoted *Z*, is given by

$$\Psi = \begin{bmatrix} x_0(\omega_k) & x_{-1}(\omega_k) & x_{-2}(\omega_k) & \cdots & x_{-N}(\omega_k) \\ x_1(\omega_k) & x_0(\omega_k) & x_{-1}(\omega_k) & \cdots & x_{-(N-1)}(\omega_k) \\ x_2(\omega_k) & x_1(\omega_k) & x_0(\omega_k) & \cdots & x_{-(N-2)}(\omega_k) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_N(\omega_k) & x_{N-1}(\omega_k) & x_{N-2}(\omega_k) & \cdots & x_0(\omega_k) \end{bmatrix} \quad (8)$$

The signal space of the Toeplitz data matrix in (8) has full rank which implies that all the incident sources can be detected whether they are coherent or non-coherent. The second step of the proposed method is to calculate the R data matrix or Q data matrix from the received Toeplitz data matrix (8) to estimate the DOA for form coherent multiple wideband sources. By using the Toeplitz data matrix in (8) the necessary information about the noise subspace or the signal subspace can be extracted using QR factorization. One of the reasons that QR factorization [15-17] is widely used in adaptive applications is that in RRQR the signal information can be effectively updated making the algorithm suitable for tracking moving sources. In addition, the computation of QR for the $n\times n$ Toeplitz matrix [13-14] requires $O(n^2)$ operations. In the proposed method, we calculate the triangular factorization R to find the DOA for the wideband signal. Using the shift invariance property of Toeplitz matrix we can partition the data matrix $\Psi$ in two ways:

$$\Psi = \begin{bmatrix} x_0(\omega_k) & y^T \\ z & F_{-1} \end{bmatrix} \quad (9)$$

$$\text{or } \Psi = \begin{bmatrix} F_{-1} & y^K \\ z^{KT} & x_0(\omega_k) \end{bmatrix} \quad (10),$$

where $F_{-1}$ is an (n-1)x(n-1) submatrix of $\Psi$,

$z^T = [x_1(\omega_k) \;\; x_2(\omega_k) \;\; \cdots \;\; x_N(\omega_k)]$, $z^{KT} = [x_N(\omega_k) \;\; \cdots \;\; x_1(\omega_k)]$,
$y^T = [x_{-1}(\omega_k) \;\; \cdots \;\; x_{-N}(\omega_k)]$, an $y^{KT} = [x_{-N}(\omega_k) \;\; \cdots \;\; x_{-1}(\omega_k)]$

Let R be the upper triangular factor of the Cholesky factorization of $\Psi^T\Psi$,

$$R^T R = \Psi^T \Psi \quad (11),$$




DOI: 01.IJCOM.02.03.32

where R is the upper triangular matrix from the QR decomposition of $\Psi$. The matrix R can be partitioned in two different ways

$$R = \begin{bmatrix} r_{1,1} & r_{fr} \\ O & R_b \end{bmatrix} \tag{12}$$

or $$R = \begin{bmatrix} R_t & r_{1c} \\ O & r_{N,N} \end{bmatrix} \tag{13},$$

where $r_{fr} = \lfloor r_{1,2} \quad \cdots \quad r_{1,N} \rfloor$, $r_{1c} = \lfloor r_{1,N} \quad \cdots \quad r_{N-1,N} \rfloor$; $R_b$ is the bottom submatrix and $R_t$ is the top submatrix have a dimension $(N-1)\times(N-1)$. From equations (9), (11) and (12) we get

$$\begin{bmatrix} r_{1,1}^2 & r_{1,1}r_{fr} \\ r_{fr}^T r_{1,1} & r_{fr}^T r_{fr} + R_b^T R_b \end{bmatrix} = \begin{bmatrix} [x_0(\omega_k)]^2 + z^T z & x_0(\omega_k)y^T + z^T T_{-1} \\ x_0(\omega_k)y + T_{-1}^T z & yy^T + T_{-1}^T T_{-1} \end{bmatrix} \tag{14}.$$

Similarly, from equations (10), (11) and (13) we get

$$\begin{bmatrix} R_t^T R_t & R_t^T r_{1c} \\ r_{1c}^T R_t & r_{1c}^T r_{1c} + r_{n,n}^2 \end{bmatrix} = \begin{bmatrix} T_{-1}^T T_{-1} + z^K z^{KT} & z^K x_0(\omega_k) + T^T y^R \\ y^{RT} T - 1 + x_0(\omega_k) & y^{RT} y^R + [x_0(\omega_k)]^2 \end{bmatrix} \tag{15}$$

Comparing the left upper submatrices in (15) we get

$$R_t^T R_t = T_{-1}^T T_{-1} + z^K z^{KT} \tag{16};$$

while comparison of the lower right matrices on both sides of (14) gives us

$$r_{fr}^T r_{fr} + R_b^T R_b = yy^T + T_{-1}^T T_{-1} \tag{17}.$$

Finally, (16) and (17) give the main relation

$$R_b^T R_b = R_t^T R_t + yy^T - z^K z^{KT} - r_{fr}^T r_{fr} \tag{18}$$

where from (14), $r_{fr}$ is given by

$$r_{fr} = \frac{x_0(\omega_k)y^T + z^T T_{-1}}{\left([x_0(\omega_k)]^2 + z^T z\right)^{1/2}} \tag{19}$$

Equations (18) and (19) represent the base in calculating the matrix R. For more detailed information on obtaining the elements of R matrix return to [13] and [14]. After the matrix R is calculated, the DOA of the wideband signal can be found, as discussed in the following paragraph. Now the calculated data matrix R with dimensions $(N+1)\times(N+1)$ can be partitioned as

$$R = \begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{bmatrix} \tag{20},$$

where $R_{11}$ is $(L\times L)$; $R_{12}$ is $L\times(N+1-L)$; and $R_{22}$ is $(N-L+1)\times(N-L+1)$. The QR factorization in (34) is called a rank-revealing QR factorization if $R_{22}$ has a small norm.

Since $R_{22}$ is very small, the basis of the noise space can be obtained from the above *R* factor as follows. Let $V$ be a permutation matrix which represents the row and column interchange. Then, given the small norm of the matrix $R_{22}$, the upper triangular matrix *R* can be written as

$$R = [R_{11} \quad R_{12}]V^T \tag{21},$$

where the matrix R in (21) defines the null space of *Y*. Let G represent any vector in the null space of *R*, i.e. $RG = 0$. To find the structure of *G*, we partition *G* into $g_1$ with *L* components and $g_2$ with *(N-L+1)* components. Then *GR*=0 implies that

$$[R_{11} \quad R_{12}] \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = 0 \tag{22}$$

so that $R_{11}g_1 + R_{12}g_2 = 0$. Since $R_{11}$ is a non-singular matrix, $g_1$ can be written in terms of $g_2$ as follows

$$g_1 = -R_{11}^{-1}R_{12}g_2 \tag{23}.$$

Then G can be written as

$$G = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} -R_{11}^{-1}R_{12} \\ I_{N-L+1} \end{bmatrix} g_2 \tag{24}.$$

To find the basis of null space *R*, we choose any set of *N-L+1* linearly independent vectors; for example, the columns of the *N-L+1* dimensional identity matrix. The basis for the null space of the upper triangular matrix *R*, which is also the null space of *Y*, is therefore given by

$$H_{qr} = \begin{bmatrix} -R_{11}^{-1}R_{12} \\ I_{N-L+1} \end{bmatrix} \tag{25}.$$

It is important to observe that the columns of the null space of $H_{qr}$ are not orthonormal. This is in contrast to the null space which can be derived from the SVD or EVD techniques.

We now note that the subspace spanned by the columns of $H_{qr}$ is orthogonal to the subspace spanned by the columns of *B*, where the columns of *B* contain the information about the AOAs of incident sources. This is similar to the well-known MUSIC algorithm in which the eigenvector of the noise subspace is orthogonal to the steering vector of the signal subspace. To find the AOAs, we search for the minimum peaks of $\left\| H_{qr}^H B(\theta) \right\|$. Since the basis of is not orthonormal, we use the orthogonal projection onto this subspace in order to improve the performance by making the basis of the null space of orthonormal. So the columns of the null space of become orthonormal as follows

$$H_O = H_{qr}\left(H_{qr}^H H_{qr}\right)^{-1} H_{qr}^H \tag{26}.$$




DOI: 01.IJCOM.02.03.32

We now employ the root MUSIC algorithm to find the wideband DOAs of the incident sources. The proposed QRC-ESPRIT using root MUSIC [18]can be written as

$$P^{-1} = a(\omega_k, \theta)^H H_o H_o^H a(\omega_k, \theta) \tag{27}$$

The roots of the polynomial are obtained from (27) for all the frequency bins k, and then they are averaged to derive the final DOA of the incident signals

$$\hat{\theta}(k) = \frac{1}{K} \sum_{k=k_L}^{k_H} \theta(k) \tag{28}$$

where $[k_L, k_H]$ denotes the index of frequency ranges which satisfy $K = k_H - k_L + 1$.

## III. SIMULATION RESULTS

Performance of the proposed method is compared with that of the TLS-ESPRIT and Root MMUSIC with spatial smoothing, which are considered one of the best DOA estimation algorithms. The specifications of the simulation are shown in Table I.

TABLE I.
SPECIFICATIONS OF SIMULATION

| | |
|---|---|
| Array configuration | 11-elements(uniform linear array) |
| Number of coherent incident signals | 2 |
| DOAs of incident signals | $30^\circ$ and $45^\circ$ |
| Modulation type | CDMA |
| Data length | 1024 |
| Signal bandwidth | 10 MHz |
| Carrier Frequency | 2 GHz |
| SNR range | 0-20 dB |
| Sampling frequency | 20 MHz |
| DFT points | 256 |
| Frequency range | 1KHz to 10 MHz |
| Number of trials | 100 |

Figures 1 and 2 show the root-mean-square-error (RMSE) in degree for Wideband DOA estimation error versus SNR form 0 to 20 dB for two coherent sources at 30° and 45°. It is observed that the proposed methods result in lower RMSE compared to the TLS-ESPRIT and Root MUSIC with spatial smoothing. In addition, the proposed method provides low complexity and computational cost $O(n^2)$ compared to the method in [11] which requires EVD or SVD and $O(n^3)$ operation. Furthermore, the proposed method does not require spatial smoothing techniques to estimate the coherent sources; whereas, TLS-ESPRIT and Root MUSIC do. In Table II, the computational cost of the proposed method is evaluated in comparison with TLS-ESPRIT and Root-MUSIC methods. Note that the unit of computation time is in seconds. We observe that the proposed method requires smaller computational cost compared with TLS-ESPRIT and Root-MUSIC.

TABLE II.
COMPUTIONAL COST

| Data length | TLS-ESPRIT with spatial smoothing | Root MUSIC with spatial smoothing | Proposed Method |
|---|---|---|---|
| 256 | 0.030 | 0.050 | 0.015 |
| 512 | 0.055 | 0.095 | 0.031 |
| 1024 | 0.104 | 0.178 | 0.053 |

## IV. CONCLUSION

A new approach is introduced for the estimation of DOA for wideband signals. The observation output is arranged into a Toeplitz matrix which enables us to perform the estimation using one or more snapshots and estimates the coherent source without any additional processing compared to TLS-ESPRIT and Root MUSIC that need spatial smoothing techniques. The proposed methods employ QR factorization to estimate an R matrix or Q matrix which has a much lower complexity and computational cost $O(n^2)$ compared to existing methods, which require EVD or SVD and $O(n^3)$ operation. Through simulations we have shown that the proposed method exhibit significant performance improvement over many subarray methods such as TLS-ESPRIT and Root MUSIC. These advantages make our proposed method appropriate for real-time implementation.

## REFERENCES


[1] J. Krim and M. Viberg. “Two decades of array signal processing research: the parametric approach.” IEEE signal Processing. Mag. Vol.13:3, pp. 67-94. Jul.1996.

[2] J. C. Chen, K. Yao, and R. E. Hudson. “Source Localization and beamforming.” IEEE signal Processing. Mag. Vol.19:2, pp. 30-39. Mar. 2002.

[3] R. O. Schmidt. “Multiple Emitter Location and Signal Parameter Estimation.” IEEE Trans. Antennas and Propagation. Vol. 34, pp. 276-280. Mar.1986.

[4] R. Roy and T. Kailath. “ESPRIT—Estimation of Signal Parameters via Rotational Invariance Techniques.” IEEE Trans. Acoust., Speech, and Signal Processing. Vol.37:7, pp.984-995. July 1989.

[5] I.Ziskind and M. Wax. “Maximum likelihood localization of multiple sources by alternating projection.” IEEE Trans. Acoustic., Speech, Signal Process. Vol.36, pp.1553-1560. Oct. 1988.

[6] M. A. Doron, A. J.Weiss, and H. Messer. “Maximum-likelihood direction finding of wide-band sources.” IEEE Trans. Signal Process. Vol. 41:1, pp. 411–414. Jan. 1993.

[7] J. C. Chen, R. E. Hudson, and K. Yao. “Maximum-likelihood source localization and unknown sensor locationestimation for wideband signals in the near-field.” IEEE Trans. Signal Process. Vol. 50:8, pp. 1843–1854. Aug. 2002.

[8] W.J. Zeng and Xi. Lin Li. “High-Resolution Multiple Wideband and Nonstationary Source Localization with Unknown Number of Sources. IEEE Trans. Signal Process. Vol. 58:6, pp. 3125–3136. Jun. 2010.

[9] H. Wang and M. Kaveh. “Coherent signal-subspace processing for the detection and estimation of angles of arrival of multiple wideband sources.” IEEE Trans. Acoust., Speech, Signal Process. Vol. ASSP-33, pp. 823–831. Aug. 1985.

[10] S. Valaee and P. Kabal. “Wideband array processing using a two-sided correlation transformation.” IEEE Trans. Signal Process. Vol. 43, pp. 160–172. Jan. 1995.

[11] Y. Yoon, L. M. Kaplan, and J. H. McClellan. “TOPS: New DOA Estimation for Wideband Signals.” IEEE Trans. Signal Processing, Vol. 54:6, pp. 1977-1988. June 2006.

[12] H. Yu and J. L. Huang. “A new Method for wideband DOA Estimation.” Wireless communication, Networking and Mobile Computing 2007. WiCom 2007, pp. 598-601. Sep. 2007.

[13] D. R. Sweet. “Fast Toeplitz orthogonalization.” Numer. Math. Vol. 53, pp. 1-21. 1984.



DOI: 01.IJCOM.02.03.32

[14] A.W. Bojanczyk, R. P. Brent, and F. R. de Hoog. "QR Factorization of Toeplitz matrices." Numer. Math. Vol. 49, pp. 81-94. 1986.

[15] C. H. Bischof and G. M. Shroff. "On updating signal subspaces." IEEE Transactions on Acoustics, Speech, and Signal Processing. Vol. 40, pp. 96-105. Jan 1992.

[16] M. P. Fargues and M. P. Ferreira. "Investigations in the numerical behavior of the adaptive rank-revealing QR factorization." IEEE Transactions on Acoustics, Speech, and Signal Processing. Vol. 43, pp. 2787-2791. Nov. 1995.

[17] M. Bouri and S. Bourennane. "High resolution methods based on rank revealing triangular factorization." Transactions on Engineering, Computing and Technology. Vol. 2, pp. 35-38. Dec 2004.

[18] A. J. Barabel. "Improved the resolution performance of eigenstructure-based direction-finding algorithm." In Proc. IEEE ICASSP 83, pp. 336-339. 1983.

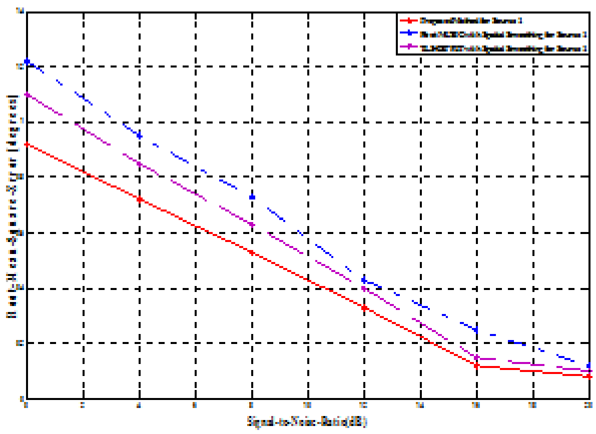


Figure 1. RMSE for wideband DOA angle estimation errors versus SNR for Source 1 at 30º by using the proposed method, Root MUSIC, and TLS-ESPRIT algorithms

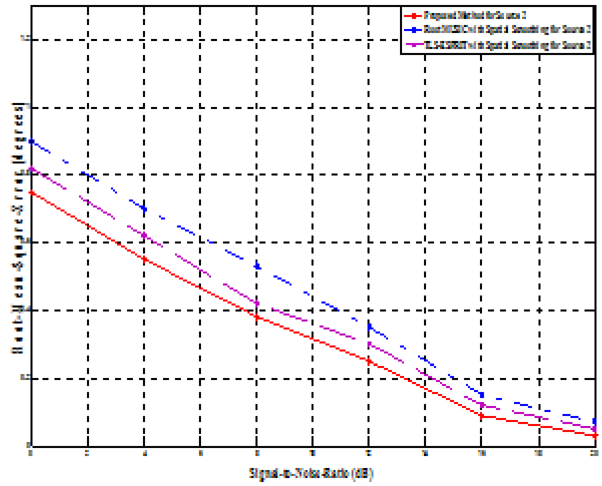


Figure 2. RMSE for wideband DOA angle estimation errors versus SNR for Source 2 at 45º by using the proposed method, Root MUSIC, and TLS-ESPRIT algorithms.




DOI: 01.IJCOM.02.03.32
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